

# Efficient compression of QRS complexes using Hermite expansion

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**Abstract**—We propose a novel algorithm for the compression of ECG signals, in particular QRS complexes. The algorithm is based on the expansion of signals with compact support into a basis of discrete Hermite functions. These functions can be constructed by sampling continuous Hermite functions at specific sampling points. They form an orthogonal basis in the underlying signal space. The proposed algorithm relies on the theory of signal models based on orthogonal polynomials. We demonstrate that the constructed discrete Hermite functions have important advantages compared to continuous Hermite functions, which have previously been suggested for the compression of QRS complexes. Our algorithm achieves higher compression ratios compared with previously reported algorithms based on continuous Hermite functions, discrete Fourier, cosine, or wavelet transforms.

**Index Terms**—QRS complex, ECG signal, compression, Hermite function, Hermite transform, signal model, orthogonal polynomials.

## I. INTRODUCTION

Some classes of electrophysiological signals have (or can be assumed to have) a compact support. These signals represent the impulse responses of a system or an organ to an electrical stimulation that is recorded on the body surface. Examples include electrocardiographic (ECG), electroencephalographic, and myoelectric signals.

The major role of electrophysiological signals is to provide information about a patient's condition, and reflect important changes in his/her state. In addition, it may also be desired to store electrophysiological signals for later analysis and reference. However, the visual analysis and monitoring of long-term repetitive signals is a tedious task that requires the presence of a human operator. In these cases, computer-based systems can be utilized to facilitate this process.

For efficient storage, automatic monitoring, and accurate interpretation of electrophysiological signals, they are usually represented by a set of features, either heuristic, such as duration and amplitude, or formal, such as the coefficients of the expansion in an orthogonal basis. In the latter case,

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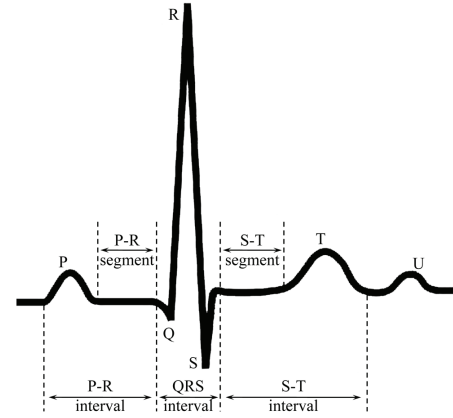


Fig. 1. The schematic structure of an ECG signal.

one can use either a continuous or a discrete basis. When a continuous basis is used, the projection and reconstruction of a compact-support signal are computed using numerical methods for integral approximation, for example, a numerical quadrature. When a discrete basis is used, a discrete signal transform, such as the discrete Fourier transform or the discrete cosine transform, is applied to a digitized signal. These signals can be obtained from continuous ones by sampling at specific sampling points. The choice of the quadrature formula, or the sampling points is an important characteristic of the representation scheme.

In both cases of continuous and discrete bases, usually only a few projection coefficients are used for the storage and reconstruction of a signal. This reduction of expansion coefficients leads to a reconstruction error. The goal of the compression optimization then is to minimize the error while maximizing the compression ratio. This can be achieved, for example, by using only the coefficients with the largest magnitude, and minimizing the number of coefficients used for reconstruction.

In this paper, we introduce a novel algorithm for the compression of *QRS complexes* (also known as QRS intervals), which are the most characteristic waves of ECG signals. The structure of an ECG signal is shown in Fig. 1. The morphology of QRS complexes is important to cardiologists on different stages of diagnosis and treatment [1]. Examples include, but are not limited to,

- 1) The detection of the rhythm origin (supraventricular, from the upper chambers of the heart, or ventricular, from the bottom chambers). This is a key factor in the

choice of the treatment.

- 2) The detection of conduction abnormalities between atria and ventricles (bundle branch blocks, fascicular blocks, and others).
- 3) The detection of accessory pathways (Wolf-Parkinson-White syndrome).
- 4) The evaluation of the effect of specific anti-arrhythmic medications (such as sodium channel blockers).

Our proposed algorithm is based on the expansion of QRS complexes into the basis of *discrete* Hermite functions. Such functions are obtained by sampling *continuous* Hermite functions at specific sampling points, not necessarily located on a uniform grid. We originally proposed this compression method in [2]. In this paper, we extend our previous work by rigorously formulating and describing the signal model used for the description of QRS complexes. In particular, we utilize results from our recently developed theory of signal models based on orthogonal polynomials [3]–[5]. We also identify a fast algorithm for the implementation of the proposed compression method. Finally, we demonstrate with experimental results that the proposed method achieves higher compression ratios, compared to other methods, when used to obtain medically acceptable compressed ECG signals. Here, we call a compressed ECG signal *medically acceptable*, if visually it is sufficiently similar to the original signal, and would not lead to an incorrect interpretation and diagnosis. In our experiments, the identification of medically acceptable compressed signals has been performed by one of the authors who is a cardiologist with extensive experience in cardiac electrophysiology.

**Related work.** Previous work on the compression of QRS complexes includes the use of continuous Hermite functions as the expansion basis [6]–[9]. Due to the shape similarity between continuous Hermite functions and QRS complexes, these functions were identified as a suitable basis for the representation and compression of the latter. These works, however, only provide a theoretical framework for the compression algorithm. The actual implementation is not discussed and no experimental data is provided that would confirm that the proposed compression method indeed performs better than other methods. In addition, as we discuss in Section II, the computer implementations of the previously proposed algorithms suffer from certain limitations, such as the inability to obtain an exact reconstruction of a signal, large computational cost, and an a priori selection of coefficients for reconstruction.

## II. BACKGROUND

In this section, we discuss the expansion of continuous signals using Hermite functions, its digital implementation, and its use in signal compression.

### A. Continuous Hermite functions

Consider the family of polynomials  $H_\ell(t)$ ,  $\ell \geq 0$ , that satisfy the recursion

$$H_\ell(t) = 2tH_{\ell-1}(t) - 2(\ell-1)H_{\ell-2}(t), \quad (1)$$

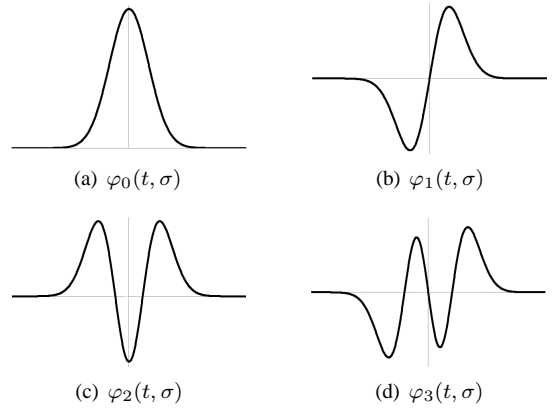


Fig. 2. First four Hermite functions (plotted for the same scale  $\sigma$ ).

for  $\ell \geq 2$ , with  $H_0(t) = 1$  and  $H_1(t) = 2t$ . They are known as *Hermite polynomials*. These polynomials are orthogonal on the real line  $\mathbb{R}$  with respect to the weight function  $e^{-t^2}$ :

$$\int_{-\infty}^{\infty} H_\ell(t)H_m(t)e^{-t^2} dt = 2^\ell \ell! \sqrt{\pi} \cdot \delta_{\ell-m}. \quad (2)$$

It immediately follows from (2) that the functions

$$\varphi_\ell(t, \sigma) = \frac{1}{\sqrt{\sigma 2^\ell \ell! \sqrt{\pi}}} e^{-t^2/2\sigma^2} H_\ell(t/\sigma) \quad (3)$$

are orthonormal on  $\mathbb{R}$  with respect to the standard inner product

$$\langle \varphi_\ell(t, \sigma), \varphi_m(t, \sigma) \rangle = \int_{\mathbb{R}} \varphi_\ell(t, \sigma) \varphi_m(t, \sigma) dt = \delta_{\ell-m}. \quad (4)$$

The set of functions  $\{\varphi_\ell(t, \sigma)\}_{\ell \geq 0}$ , called *continuous Hermite functions*, is an orthonormal basis in the Hilbert space of continuous functions defined on  $\mathbb{R}$  [10]–[13]. Any such function  $s(t)$  can be represented as a linear combination of the basis functions

$$s(t) = \sum_{\ell \geq 0} c_\ell \varphi_\ell(t, \sigma), \quad (5)$$

where

$$c_\ell = \langle s(t), \varphi_\ell(t, \sigma) \rangle = \int_{\mathbb{R}} s(t) \varphi_\ell(t, \sigma) dt.$$

The first four continuous Hermite functions are shown in Fig. 2. Notice that each  $\varphi_\ell(t, \sigma)$  quickly approaches zero as the value of  $|t|$  increases: since  $H_\ell(t/\sigma)$  is a polynomial of degree  $\ell$ ,

$$\lim_{|t| \rightarrow \infty} e^{-t^2/2\sigma^2} H_\ell(t/\sigma) = 0.$$

As a consequence, for practical purposes we can assume that each continuous Hermite function has a compact support. Since in this paper we often work only with the first  $L$  continuous Hermite functions, we assume that  $\varphi_0(t, \sigma), \varphi_1(t, \sigma), \dots, \varphi_{L-1}(t, \sigma)$  have the same compact support  $[-T_\sigma, T_\sigma]$ , where  $T_\sigma$  is a suitably chosen constant that depends on  $\sigma$  and  $L$ . In other words, we assume

$$\varphi_\ell(t, \sigma) = 0 \text{ for } t \notin [-T_\sigma, T_\sigma],$$

where  $0 \leq \ell < L$ . If a signal  $s(t)$  also has a compact support

of  $[-T_\sigma, T_\sigma]$ , then we can compute the coefficients  $c_\ell$  using a finite integral:

$$c_\ell = \int_{\mathbb{R}} s(t)\varphi_\ell(t, \sigma)dt = \int_{-T_\sigma}^{T_\sigma} s(t)\varphi_\ell(t, \sigma)dt. \quad (6)$$

### B. Compression with continuous Hermite functions

**Coefficient-based compression.** In practical applications, only a finite number  $M$  of Hermite functions are used to represent the signal  $s(t)$  in (5). Accordingly, only a few coefficients  $c_{\ell_0}, \dots, c_{\ell_{M-1}}$  need to be computed. Here,  $c_{\ell_m}$  corresponds to  $\varphi_{\ell_m}(t, \sigma)$  in (6). The approximated signal then is

$$\hat{s}(t) = \sum_{m=0}^{M-1} c_{\ell_m} \varphi_{\ell_m}(t, \sigma). \quad (7)$$

Alternatively, a larger pool of coefficients can be computed, from which  $M$  optimal ones are selected. It is well-known that for an orthonormal basis selecting coefficients with the largest magnitude minimizes the approximation error computed as the energy of the difference between the signal  $s(t)$  and its approximation with  $M$  basis functions. If a basis is orthogonal, but not orthonormal (i.e. the basis functions do not have unit norms), an additional weighting of coefficients proportionally to the norms of the basis functions can be performed.

**Digital implementation.** The coefficients  $c_\ell$  in (6) and the Hermite expansion (5) are computed using the continuous functions. However, in practice they have to be computed in a digital form.

In particular, for each coefficient  $c_{\ell_m}$  the integral in (6) can be calculated using a numerical quadrature that is based, for example, on a rectangle rule:

$$\begin{aligned} c_{\ell_m} &= \int_{-T_\sigma}^{T_\sigma} s(t)\varphi_{\ell_m}(t, \sigma)dt \\ &\approx \sum_{k=-K}^K s(\tau_k)\varphi_{\ell_m}(\tau_k, \sigma)(t_k - t_{k-1}). \end{aligned} \quad (8)$$

Here,  $-T = t_{-K-1} < t_{-K} < \dots < t_{K-1} < t_K = T$ . Each sampling point  $\tau_k$  is located inside the corresponding interval:  $t_{k-1} \leq \tau_k \leq t_k$ .

Then, we can compute the discrete version of the approximation signal  $\hat{s}(t)$ . It corresponds to  $\hat{s}(t)$  sampled at points  $\tau_k$ ,  $-K \leq k \leq K$ :

$$\hat{s}(\tau_k) = \sum_{m=0}^{M-1} c_{\ell_m} \varphi_{\ell_m}(\tau_k, \sigma). \quad (9)$$

Usually, points  $t_k$  are assumed to lie on a uniform grid, such that  $t_k - t_{k-1} = \Delta$  for all  $k$ . Then (8) and (9) can be expressed in matrix-vector notation. Let us define vectors

$$\mathbf{s} = \begin{pmatrix} s(\tau_{-K}) \\ \vdots \\ s(\tau_K) \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} c_0 \\ \vdots \\ c_{M-1} \end{pmatrix}, \quad \hat{\mathbf{s}} = \begin{pmatrix} \hat{s}(\tau_{-K}) \\ \vdots \\ \hat{s}(\tau_K) \end{pmatrix}. \quad (10)$$

Then

$$\mathbf{c} = \Delta \Phi^T \mathbf{s}, \quad (11)$$

$$\hat{\mathbf{s}} = \Phi \mathbf{c}, \quad (12)$$

where  $\Phi \in \mathbb{R}^{(2K+1) \times M}$ , such that its  $m$ -th column is the  $\ell_m$ -th Hermite function sampled at the points  $\tau_{-K}, \tau_{-K+1}, \dots, \tau_K$ :

$$\Phi = \left[ \varphi_{\ell_m}(\tau_{k-K}, \sigma) \right]_{0 \leq k < 2K+1, 0 \leq m < M}. \quad (13)$$

As follows from (11)-(12), in order to achieve the perfect reconstruction  $\hat{\mathbf{s}} = \mathbf{s}$ ,  $\Phi$  must satisfy  $\Phi \Phi^T = I_{2K+1}$ .

**Compression of QRS complexes: Previous work.** The compression of QRS complexes using the expansion into continuous Hermite functions has been studied in [6]–[9]. It was originally motivated by the visual similarity of QRS complexes, centered around their peaks, and Hermite functions, as can be observed from Figs. 1 and 2. Varying the value of  $\sigma$  corresponds to “stretching” or “compressing” Hermite functions  $\varphi_\ell(t, \sigma)$  to optimally match a given QRS complex.

Since ECG signals are usually available as discrete signals equidistantly sampled at  $\tau_k = k\Delta$ , previously reported work assumed, explicitly or implicitly,  $t_k = \tau_k = k\Delta$  in (8)–(9), and hence led exactly to the matrix-vector products in (11) and (12). In addition, they proposed to use only the *first*  $M$  Hermite functions  $\varphi_0(t, \sigma), \dots, \varphi_{M-1}(t, \sigma)$  for the approximation of QRS complexes.

This compression algorithm has several important limitations. First of all, since  $\Phi \Phi^T \neq I_{2K+1}$  for  $\tau_k = k\Delta$ , the approximation  $\hat{\mathbf{s}}$  does not converge to original signal  $\mathbf{s}$ , regardless of the number  $M$  of Hermite functions used for the construction of an approximation. As a result, original signal  $\mathbf{s}$  cannot be reconstructed exactly.

This problem could theoretically be addressed by using  $M = 2K + 1$  coefficients and replacing  $\Phi^T$  with  $\Phi^{-1}$  to compute coefficients  $\mathbf{c}$  in (11). However, the computation of  $\Phi^{-1}$  is a non-trivial task. Moreover, the matrix-vector product  $\Phi^{-1} \mathbf{s}$  requires  $(2K + 1)^2$  operations. This cost can become prohibitive for large value of  $K$  and make this approach impractical.

Finally, the approximation of  $\hat{s}(t_k)$  with the *first*  $M < 2K + 1$  coefficients  $c_0, c_1, \dots, c_{M-1}$  in (8) may not be the optimal choice for the construction of approximation  $\hat{\mathbf{s}}$  with  $M$  basis functions.

In Section IV, we propose an improved compression algorithm that samples ECG signals at non-equidistant points, and selects coefficients  $c_{\ell_m}$  with the *largest* magnitude among a larger selection of coefficients.

### III. SIGNAL MODEL FOR QRS COMPLEXES

In this section, we construct a signal model for the description of QRS complexes. It is based on the scaled Hermite polynomials. We identify the Fourier transform for this model, called a *Hermite transform*, as well as its inverse. We also describe a fast algorithm for the corresponding Hermite transform. These results will be used in Section IV to construct and implement the novel compression algorithm.

#### A. Signal model

Let us define polynomials

$$P_\ell(t) = \frac{1}{\sqrt{2^\ell \ell!}} H_\ell(t), \quad (14)$$

where  $H_\ell(t)$  denotes the  $\ell$ -th Hermite polynomials (1). Throughout the paper, we refer to polynomials  $P_\ell(t)$ ,  $\ell \geq 0$ , as *scaled Hermite polynomials*.

**Signal model.** Consider the vector space  $\mathcal{M}$  of functions spanned by the first  $N$  scaled Hermite polynomials (14):

$$\mathcal{M} = \left\{ s_0 P_0(t) + s_1 P_1(t) + \dots + s_{N-1} P_{N-1}(t) \right\}, \quad (15)$$

where  $(s_0, s_1, \dots, s_{N-1})^T \in \mathbb{R}^N$ . Since each  $P_\ell(t)$  is a polynomial of degree  $\ell$ ,  $\mathcal{M}$  is a vector space of degree  $N$ . It is closed under the addition and linear scaling of its elements.

Assume that  $\mathcal{M}$  is also closed under the multiplication of its elements modulo polynomial  $P_N(t)$ <sup>1</sup>. As we demonstrate in [3]–[5], in this case  $\mathcal{M}$  has exactly  $N$  spectral components, and the  $k$ -th component of a signal

$$s(t) = s_0 P_0(t) + s_1 P_1(t) + \dots + s_{N-1} P_{N-1}(t)$$

is defined as

$$s(\alpha_k) = s_0 P_0(\alpha_k) + s_1 P_1(\alpha_k) + \dots + s_{N-1} P_{N-1}(\alpha_k).$$

where  $\alpha_0, \alpha_1, \dots, \alpha_{N-1}$  are the roots of  $P_N(t)$ . Note that  $\alpha_0, \alpha_1, \dots, \alpha_{N-1}$  are all distinct real numbers [10]. Without loss of generality, we assume  $\alpha_0 < \alpha_1 < \dots < \alpha_{N-1}$ .

**Hermite transform.** The spectral decomposition of signal  $s(t)$  is defined as

$$S(\alpha) = (s(\alpha_0) \quad s(\alpha_1) \quad \dots \quad s(\alpha_{N-1}))^T. \quad (16)$$

We can express the spectral decomposition of  $s(t)$  in the following matrix-vector notation. Consider the  $N \times N$  matrix

$$\mathcal{P}_{P,\alpha} = \left[ P_\ell(\alpha_k) \right]_{0 \leq k, \ell < N}. \quad (17)$$

The  $\ell$ -th column of  $\mathcal{P}_{P,\alpha}$  is polynomials  $P_\ell(t)$  evaluated at  $\alpha_0, \dots, \alpha_{N-1}$ . In general, matrices with this structure are called *polynomial transforms*. In the particular case of (17), We call  $\mathcal{P}_{P,\alpha}$  the (*forward*) *Hermite transform*<sup>2</sup>.

The spectral decomposition (16) can then be computed as

<sup>1</sup>Strictly speaking, we must consider two separate spaces  $\mathcal{A}$  and  $\mathcal{M}$  that have the same elements:  $\mathcal{A} = \mathcal{M}$ .  $\mathcal{A}$  is an algebra: it is a vector space closed under the addition and multiplication of its elements modulo  $P_N(t)$ . Then  $\mathcal{M}$  can be viewed as an  $\mathcal{A}$ -module, and its spectral decomposition can be defined.

$\mathcal{A}$  and  $\mathcal{M}$  comprise a part of what we call an *algebraic signal model*. It is a central component of the *algebraic signal processing theory* developed by the authors in [3]–[5], [14]–[18].

In particular, in [3]–[5] we introduce signal models based on orthogonal polynomials (of which Hermite polynomials are a special case). A family of polynomials  $\{P_\ell(t)\}_{\ell \geq 0}$  is called *orthogonal*, if they satisfy a recursion of the form  $tP_\ell(t) = a_\ell P_{\ell-1}(t) + b_\ell P_\ell(t) + c_\ell P_{\ell+1}(t)$ , usually with initial conditions  $P_0(t) = 1$  and  $P_{-1} = 0$ . Each family is orthogonal over an interval  $I \subseteq \mathbb{R}$  with a weight function  $w(t) : \int_I P_\ell(t) P_m(t) w(t) dt = 0$  if  $\ell \neq m$ . Each polynomial  $P_\ell(t)$  has exactly  $\ell$  simple real roots. A detailed discussion on orthogonal polynomials can be found in [10]–[13].

<sup>2</sup>Hermite transform have been defined previously in other contexts. In particular, one should not confuse the Hermite transform (17) with the Hermite transform defined in [19], [20], which, to a large extent, corresponds to our definition of the inverse Hermite transform (19).

the matrix-vector product

$$\begin{pmatrix} s(\alpha_0) \\ s(\alpha_1) \\ \vdots \\ s(\alpha_{N-1}) \end{pmatrix} = \mathcal{P}_{P,\alpha} \cdot \begin{pmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-1} \end{pmatrix}. \quad (18)$$

**Inverse Hermite transform.** In general, it is non-trivial to compute the inverse of matrix  $\mathcal{P}_{P,\alpha}$  for arbitrary polynomials  $P_\ell(t)$ . However, we demonstrated in [3]–[5] that the inverse of the Hermite transform (17) can be found as

$$\mathcal{P}_{P,\alpha}^{-1} = \mathcal{P}_{P,\alpha}^T \cdot D, \quad (19)$$

where  $D \in \mathbb{R}^{N \times N}$  is a diagonal matrix with  $k$ -th diagonal element equal to

$$D_{k,k} = \frac{\sqrt{2/N}}{P_{N-1}(\alpha_k) P'_N(\alpha_k)}.$$

### B. Fast algorithm for Hermite transform

A straightforward computation of the matrix-vector product in (18) requires, in general,  $2N^2$  additions and multiplications. Alternatively,  $43N \log_2^2 N$  additions and multiplications can be required, if one uses an algorithm proposed in [21]. In both cases, the computational cost can become unacceptable for large values of  $N$ , especially if the product has to be computed in real-time (for example, in the case of ECG signal processing). The same applies to the inverse  $\mathcal{P}_{H,\alpha}^{-1}$ .

Here, we provide an improved computational algorithm for the matrix-vector product in (18). It reduces the computational cost approximately by a factor of 4. We only state the algorithm here, without the proofs. It can be verified by direct computation. However, originally we derived this algorithm by applying the theory of fast polynomial transforms developed in [4], [22] to the signal space (15).

**Theorem 1** *Let  $N = 2M$  be an even number. Then the polynomial transform (17) can be factored as*

$$\begin{aligned} \mathcal{P}_{P,\alpha} &= \begin{pmatrix} J_M & -J_M \\ I_M & I_M \end{pmatrix} \begin{pmatrix} I_M & & & \\ & \alpha_M & & \\ & & \ddots & \\ & & & \alpha_{N-1} \end{pmatrix} \\ &\times \begin{pmatrix} \mathcal{P}_M & \\ & \mathcal{P}_M \end{pmatrix} B_0 B_1 \dots B_{M-1} L_2^N. \end{aligned} \quad (20)$$

Here,  $I_M$  is an identity matrix of size  $M$ , and  $J_M$  is a complimentary identity matrix of size  $M$ :

$$J_M = \begin{pmatrix} & & & 1 \\ & & \ddots & \\ & & & 1 \\ 1 & & & \end{pmatrix}.$$

Matrix  $\mathcal{P}_M$  is an  $M \times M$  polynomial transform defined as

$$\mathcal{P}_M = \left[ P_{2\ell}(\alpha_{k+M}) \right]_{0 \leq k, \ell < M}.$$

Each  $B_k$  is an identity matrix except its  $(M+k, M+k)$ -th and  $(M+k-1, M+k)$ -th elements are equal to  $\sqrt{2/(2k+1)}$  and

$-\sqrt{2k/(2k+1)}$ , respectively.  $L_2^N$  is a permutation matrix: its  $(k, \ell)$ -th element is 1 if  $\ell = \lfloor \frac{2k(N+1)}{N} \rfloor \bmod N$ , and 0 otherwise.

The algorithm for odd values of  $N$  is similar.

**Theorem 2** Let  $N = 2M + 1$  be an odd number. Then the polynomial transform (17) can be factored as

$$\mathcal{P}_{P,\alpha} = \begin{pmatrix} J_M & -J_M \\ 1 & I_M \end{pmatrix} \begin{pmatrix} I_{M+1} & & & \\ & \alpha_{M+1} & & \\ & & \ddots & \\ & & & \alpha_{2M} \end{pmatrix} \times \begin{pmatrix} \mathcal{P}_{M+1} & \\ & \mathcal{P}_M \end{pmatrix} B_0 B_1 \dots B_{M-1} L_2^N. \quad (21)$$

Here,  $\mathcal{P}_{M+1}$  is an  $(M+1) \times (M+1)$  polynomial transform defined as

$$\mathcal{P}_{M+1} = \left[ P_{2\ell}(\alpha_{k+M}) \right]_{0 \leq k, \ell < M+1}.$$

$\mathcal{P}_M$  is a submatrix of  $\mathcal{P}_{M+1}$  obtained by removing the first row and last column of the latter. Each  $B_k$  is an identity matrix except its  $(M+1+k, M+1+k)$ -th and  $(M+k, M+1+k)$ -th elements are equal to  $\sqrt{2/(2k+1)}$  and  $-\sqrt{2k/(2k+1)}$ , respectively. Matrices  $I_M$ ,  $J_M$ , and  $L_2^N$  are as described above.

We show in [4] that the polynomial transforms  $\mathcal{P}_M$  and  $\mathcal{P}_{M+1}$  in Theorems 1 and 2 are also based on orthogonal polynomials. Hence, they can also be computed using approximately  $2M^2$  or  $43M \log_2^2 M$  additions and multiplications, depending on the choice of the algorithm. Since other matrices in factorizations (20) and (21) require approximately  $3N$  additions and multiplication, the computational algorithms in Theorems 1 and 2 require approximately four times fewer operations compared to other algorithms.

#### IV. COMPRESSION ALGORITHM

As we mentioned in Section II, the parameter  $\sigma$  in the definition of the continuous Hermite functions (3) can be used to “stretch” and “compress” the functions relatively to the signal  $s(t)$ .

**Algorithm modifications.** Alternatively, we can fix  $\sigma = 1$ , and introduce a parameter  $\lambda$  to “stretch” and “compress” signal  $s(t\lambda)$  instead. In this case the numerical quadrature (8) can be rewritten as

$$c_\ell = \int_{-T_\lambda}^{T_\lambda} s(t\lambda) \varphi_\ell(t, 1) dt \approx \sum_{k=-K}^K s(\tau_k \lambda) \varphi_\ell(\tau_k, 1) (t_k - t_{k-1}). \quad (22)$$

Furthermore, instead of sampling points on a uniform grid, we propose to use non-equispaced sampling points associated with the roots of Hermite polynomials. Namely, we set  $\tau_k = \alpha_{k+K}$ ,  $-K \leq k \leq K$ , to be the roots of the scaled Hermite polynomial  $P_{2K+1}(t)$ .

**Discrete Hermite functions.** We call the vector

$$\Phi_\ell = (\varphi_\ell(\alpha_0, 1) \quad \varphi_\ell(\alpha_1, 1) \quad \dots \quad \varphi_\ell(\alpha_{N-1}, 1))^T,$$

the  $\ell$ -th discrete Hermite function.

If, as previously,  $t_k - t_{k-1} = \Delta$  in (22), then the computation of each coefficient  $c_\ell$  in (22) can be seen as an inner product of the discrete signal

$$\mathbf{s} = (s(\alpha_0 \lambda) \quad s(\alpha_1 \lambda) \quad \dots \quad s(\alpha_{2K+1} \lambda))^T$$

with the  $\ell$ -th discrete Hermite function  $\Phi_\ell$ :

$$c_\ell \approx \Delta \sum_{k=-K}^K s(\alpha_{k+K} \lambda) \varphi_\ell(\alpha_{k+K}, 1) = \Delta \cdot \langle \Phi_\ell, \mathbf{s} \rangle.$$

**Expansion matrix.** The matrix  $\Phi$  in (13) can now be rewritten as

$$\Phi = \pi^{-1/4} W \mathcal{P}_{P,\alpha}, \quad (23)$$

where

$$W = \text{diag} (e^{-\alpha_k^2/2})_{0 \leq k < 2K+1}$$

is a diagonal matrix, and  $\mathcal{P}_{P,\alpha}$  is given in (17).

Finally, if  $M = 2K + 1$ , and  $\Phi$  is a square  $(2K + 1) \times (2K + 1)$  matrix, then it follows from (19) that the rows of  $\Phi$  form an orthogonal basis:

$$\Phi \Phi^T = \pi^{-1/2} W^2 D^{-1}. \quad (24)$$

We denote the diagonal matrix on the right-hand side of (24) as  $\Lambda$ . Later, to account for the vector norms, we will pre-multiply the input signal  $\mathbf{s}$  with the weight matrix  $\Lambda^{-1}$ .

**Proposed algorithm.** The proposed compression algorithm operates as follows. We assume that the ECG signal  $s(t)$  is sampled at points  $\alpha_{k+K} \lambda$ ,  $-K \leq k \leq K$ , to obtain a vector of samples

$$\mathbf{s} = (s(\alpha_0 \lambda), s(\alpha_1 \lambda), \dots, s(\alpha_{2K} \lambda))^T.$$

The selection of parameter  $\lambda$  is discussed in Section V. Then we compute the vector of expansion coefficients

$$\mathbf{c} = \Delta \Phi^T \Lambda^{-1} \mathbf{s},$$

where  $\Phi$  and  $\Lambda$  are given in (23) and (24). Following this, we construct the vector  $\hat{\mathbf{c}}$  by keeping only  $M$  coefficients in  $\mathbf{c}$  with the largest magnitudes and setting others to zero. Finally, we use  $\hat{\mathbf{c}}$  to obtain the signal approximation

$$\hat{\mathbf{s}} = \Delta^{-1} \Phi \hat{\mathbf{c}}. \quad (25)$$

**Advantages.** The proposed algorithm addresses several limitations of the original compression algorithms based on continuous Hermite functions.

Since the rows of  $\Phi$  form an orthogonal basis, increasing the number of vectors used for the approximation of a signal decreases the approximation error. Thus, if an exact reconstruction of signals is required, it can be achieved by using  $M = 2K + 1$  coefficients to obtain  $\hat{\mathbf{c}}$ .

Furthermore, we minimize the approximation error, since we pre-compute all coefficients  $c_0, c_1, \dots, c_{2K}$ , and only after this select the ones with the largest magnitude to obtain  $\hat{\mathbf{c}}$ .

Finally, the proposed algorithm is more efficient computa-

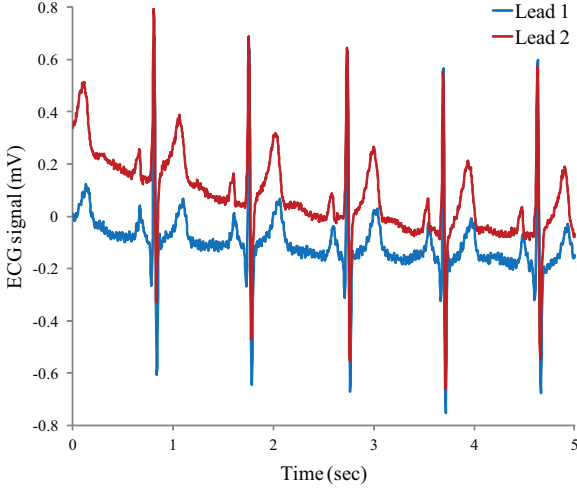


Fig. 3. An example ECG signal with two leads.

tionally, since the computational cost of  $\Phi$  in (23) (as well as  $\Phi^T$ ) is approximately 4 times lower compared to the cost of  $\Phi$  (and  $\Phi^T$ ) in (13), as explained in Section III-B.

## V. EXPERIMENTS

**Setup.** In order to analyze the performance of the proposed compression algorithm, we study the compression of QRS complexes extracted from ECG signals in the MIT-BIH ECG Compression Test Database [23]. The database contains 168 ECG signals sampled at 250 Hz. An example ECG signal with two recorded channels, or *leads*, is shown in Fig. 3.

**Preprocessing.** To make the experiments uniform across the database, we only used the first 10 seconds of the first lead of each signal. All detectable QRS complexes were extracted automatically, a total of  $N = 1486$  complexes. During the extraction, we require that each extracted complex is centered around the R peak. As a result, each extracted complex is available as a discrete signal of length  $2K + 1 \in \{27, 29, 31\}$ , where the  $(K+1)$ -th sample corresponds to the R peak. Hence, each signal represents a continuous QRS complex of duration 104, 112, or 120 milliseconds sampled at 250 Hz.

For the new compression algorithm, QRS complexes must be sampled at points  $\alpha_{k+K}\lambda$  proportional to the roots of  $P_{2K+1}(t)$ . These signals are not available directly from MIT-BIH ECG Compression Test Database. Hence, we had to construct the required discrete signals prior to running the experiments. In particular, we reconstructed continuous QRS complexes by interpolating the extracted discrete QRS complexes with *sinc* functions. Then we sampled the obtained continuous signals at the required points  $\alpha_{k+K}\lambda$ . The value of the parameter  $\lambda$  was determined for each lead separately to maximize the resulting compression ratio.

Similarly, we determined the value of the parameter  $\sigma$  for each lead to maximize the compression ratio for the original compression algorithm that expands QRS complexes into continuous Hermite functions.

In a real-time system, the parameters  $\lambda$  or  $\sigma$  would be pre-set for each lead. In addition, the incoming continuous ECG

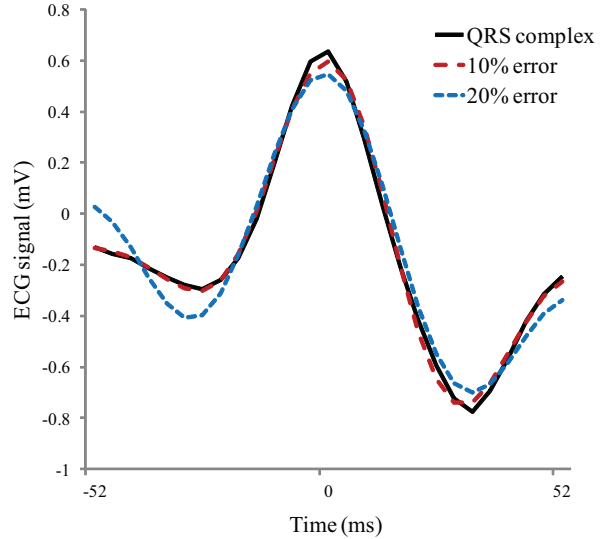


Fig. 4. A QRS complex and its approximations with 10% and 20% errors.

signal would be immediately sampled at the desired points. Hence, the interpolation step would not be required.

**Objective.** The main objective of the experiments is to achieve the maximal compression ratio for a desired approximation error. We define the error as the norm of the difference between the original signal and its approximation, normalized by the norm of the original signal:

$$\text{approximation error} = \frac{\|\hat{\mathbf{s}} - \mathbf{s}\|_2}{\|\mathbf{s}\|_2}. \quad (26)$$

In this paper, we seek to achieve a 10% approximation error. It has been verified by one of the authors who is an experienced cardiologist, that all signals in the MIT-BIH ECG Compression Database can be compressed with the methods considered in this paper with the 10% error while remaining *medically acceptable*. This means that visually all signals remain sufficiently similar to their original versions, and would not lead to incorrect interpretation and diagnoses.

We also provide compression ratios for 15%, 20%, and 25% approximation errors. In general, signals in the MIT-BIH ECG Compression Database compressed with these errors have been found medically unacceptable. For example, in our experiments for the approximation error of 20%, 630 out of 1486 (42.4%) compressed QRS complexes were identified as medically unacceptable. However, most of the distortions were introduced at the boundaries of the compressed QRS complexes, as demonstrated in the example shown in Fig. 4. Potentially, this problem could be addressed by smoothing the boundaries, and hence reducing the approximation error.

**Compression algorithms.** For the original compression algorithm that expands QRS complexes into continuous Hermite functions, as described in Section II-B, we compute a sufficient number  $M$  of coefficients  $c_0, \dots, c_{M-1}$  in (11). We use the minimal possible  $M$  that ensures that the reconstruction  $\hat{\mathbf{s}}$  in (12) has the desired approximation error (26).

For the new compression algorithm, we compute  $2K + 1$  coefficients, and select a sufficient number  $M$  of the coeffi-



Error	Proposed algorithm	Original algorithm	DFT-based	DCT-based	DWT-based
10%	5.3 (5.8)	3.5 (9.0)	3.7 (8.3)	4.3 (7.3)	3.3 (9.4)
15%	7.0 (4.4)	4.3 (7.2)	4.2 (7.4)	5.1 (6.1)	4.2 (7.5)
20%	9.2 (3.4)	5.0 (6.2)	4.6 (6.7)	5.8 (5.3)	4.8 (6.5)
25%	10.4 (2.9)	5.8 (5.4)	5.1 (6.1)	6.6 (4.7)	5.5 (5.6)

TABLE I

AVERAGE COMPRESSION RATIOS OF DIFFERENT ALGORITHMS FOR 10%, 15%, 20% AND 25% APPROXIMATION ERRORS. THE AVERAGE NUMBER OF COEFFICIENTS REQUIRED TO ACHIEVE THE DESIRED ERROR IS INDICATED IN THE PARENTHESIS NEXT TO THE CORRESPONDING RATIO.

coefficients with the largest magnitude to obtain the approximation  $\hat{s}$  in (25) that yields the required approximation error (26).

In addition, we study the accuracy of compression algorithms based on widely used orthogonal discrete signal transforms. In particular, we consider the discrete Fourier transform (DFT), the discrete cosine transform (DCT), and discrete wavelet transform (DWT). In the latter case, we consider an orthogonal DWT based on Daubechies filters of length 4 with three levels of decomposition [24]. Since all these transforms are orthogonal, we can replace  $\Phi$  in (11) and (12) with the corresponding transform, apply the transforms to the signal  $\mathbf{s}$  in (10), and select a sufficient number  $M$  of coefficients with the largest magnitudes, such that reconstruction  $\hat{s}$  in (12) has the desired approximation error (26).

**Results.** The average compression ratios for 10–25% approximation errors are shown in Table I. The ratios were computed as

$$\text{average ratio} = \frac{\sum_{n=0}^{N-1} (2K_n + 1)}{\sum_{n=0}^{N-1} M_n}.$$

Here,  $2K_n + 1 \in \{27, 29, 31\}$  is the length of the  $n$ -th sampled QRS complex, and  $M_n$  is the number of coefficients required to achieve the desired approximation error. Naturally, the higher the compression ratio, the better the algorithm performance, since fewer coefficients are required.

In Table I, we also identify the average minimal number  $M$  of coefficients  $c_{\ell_m}$  required to achieve the desired approximation error. This characteristic is included for implementation purposes and the ease of interpretation of compression ratios.

## VI. DISCUSSION AND CONCLUSIONS

As we observe from Table I, the proposed compression algorithm has the highest compression ratio for all considered approximation errors. In particular, it requires on average only 6 coefficients to reconstruct compressed QRS complexes that are medically acceptable. This is a 25% improvement compared to using DCT as the compressing transform, a 33% improvement compared to using DFT or the compression algorithm based on continuous Hermite functions, and a 40% improvement compared to using DWT. As an interesting observation, recall that here we are using the DWT with three decomposition levels; we tested DWT with other numbers of decomposition levels, and the compression ratios were even lower.

The larger compression ratio of the proposed algorithm in comparison with the original expansion into continuous Hermite functions can be explained by the improvements discussed in Sections II and IV. The comparison with other algorithms, however, yields important insights into the advantages of the proposed algorithm.

Orthogonal transforms, such as DFT, DCT, and DWT, have been widely used for signal compression, for example, in image compression standards, such as JPEG and JPEG 2000 (for the description of these standards, see, for example, the discussion in [24] and references therein). These compression methods can efficiently compress “smooth” signals by capturing the low-frequency part of a signal with a few expansion coefficients, and removing the high-frequency part that requires many additional coefficients. However, the high-frequency component of a QRS complex cannot be dropped without distorting the signal. Consequently, compression methods based on DFT, DCT, and DWT have to use a larger number of expansion coefficients.

**Future improvements.** As we mentioned in Section V, the compression with an approximation error higher than 10% frequently leads to medically unacceptable signals. However, most of the distortion is introduced at the boundaries of the compressed QRS complexes.

Fig. 4 shows an example of such distortions for the 20% approximation error. While the shape of the compressed signal is similar to the original QRS complex, the compressed signal is clearly corrupted at the left and right boundaries. As a result, discontinuities are introduced on the edge between the compressed QRS complex and the preceding P-R segment, as well as on the edge of the QRS complex and the following S-T segment. Potentially, one could address this problem by smoothing the compressed signal at the boundaries, thus reducing the approximation error.

**Conclusions.** We have constructed a novel algorithm for the compression of QRS complexes. The proposed algorithm is based on the expansion of signals with compact support (such as ECG signals) into the basis of discrete Hermite functions. These functions are constructed by sampling the continuous Hermite functions at sampling points proportional to the roots of a corresponding Hermite polynomial.

The proposed algorithm uses results from our recently developed theory of signal models for orthogonal polynomials. As confirmed by the experiments, the novel algorithm achieves a higher compression ratio compared with the original algo-

rithm based on the expansion into continuous Hermite functions, as well as other widely used compression algorithms.

In addition, we developed a fast computational algorithm for the proposed compression method. The proposed approach reduces the number of operations approximately by a factor of 4.

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